the target to the initial energy of the projectile, and the latter being the ratio of the target momentum to the initial momentum of the projectile. If these ratios are small, they can be neglected and (6) will reduce to

$$E = \tfrac{1}{2}[\alpha/(1 + \alpha)]mv_s^2$$

which is the elementary expression for the energy loss in a completely inelastic collision. Consequently,

$$W = E_0 + \tfrac{1}{2}[\alpha/(1 + \alpha)]mv_s^2 \quad (7)$$

The substitution of (7) into (5) yields

$$\frac{v_r}{v_s} = \left[\frac{1}{1+\alpha}\left(1 - \frac{\alpha}{1+\alpha} - \frac{2E_0}{mv_s^2}\right)\right]^{1/2} \quad (8)$$

Finally, if E_0 represents a small part of the impact energy, the term involving E_0 in (8) may be ignored. Such would be the case for thin targets and when v_s is well above the limiting velocity for perforation. Thus, we obtain from Eq. (8),

$$v_r/v_s = 1/(1 + \alpha)$$

which is the simple expression (1).

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Effect of Argon Addition on Shock-Layer Radiance of CO_2-N_2 Gas Mixtures

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1. Effect of Argon Addition

The uncertainties in the chemical composition of the Mars and Venus atmospheres underlying many of the studies of possible unmanned planetary missions can cause an undesirable margin of heat-shield weight to be added, which reduces the available payload and usefulness of these space probes.

Recently, the possibility of relatively large amounts of argon in the Mars atmosphere[1] required a reappraisal of the equilibrium radiative heat transfer to blunt entry bodies for these missions. As a monatomic gas, the presence of argon would be expected generally to raise the temperature and therefore the radiation of the shock layer in front of an entry body for the same atmospheric density and vehicle speed. However, in certain speed ranges, dilution of the basic mixture by argon addition will reduce total radiation from the gas if the argon emissivity itself is low. Thus, there is no clearly defined trend to changes in radiative energy transfer due to the argon addition for all flight velocities of interest.

A Jet Propulsion Laboratory thermochemistry and real-gas normal shock computer program, used previously to give solutions to the equilibrium gas radiance of a CO_2-N_2 mixture,[2] has recently been extended to cases of high argon content. This program used thermochemistry input data obtained from the General Electric Company Missile and Space

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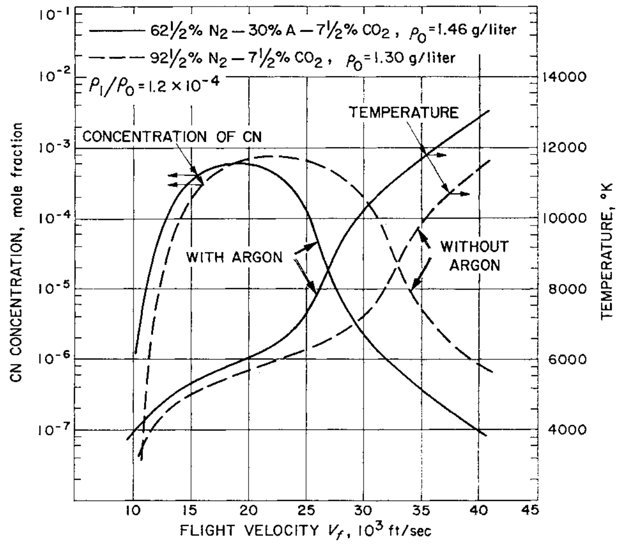

Fig. 1 Temperature and concentration of CN in mixtures with and without argon.

Vehicle Department,[3] and the radiance was computed with existing emissivity data.[4] In the lower speed range, up to 30,000 fps, the molecular band systems of the CN radical contribute the largest part of the total radiated energy for a CO_2-N_2 mixture. This was first suggested in Ref. 5 and shown in Ref. 6. With argon added, the present calculations show that the CN concentration in the gas mixture is affected such as to reduce the number of CN particles formed and to shift the peak in the concentration to lower flight velocities (Fig. 1). This reduced concentration, more than compensating for generally higher temperatures, causes the radiance of the argon mixture to fall below the comparable value for a CO_2-N_2 mixture (Fig. 2) in the relatively narrow speed range between 21,000 and 27,000 fps. Up to the latter speed, the effect of argon addition is relatively small and of varying direction compared to the higher speed range. For higher speeds, the main radiation mechanism is deionization and electron ac-

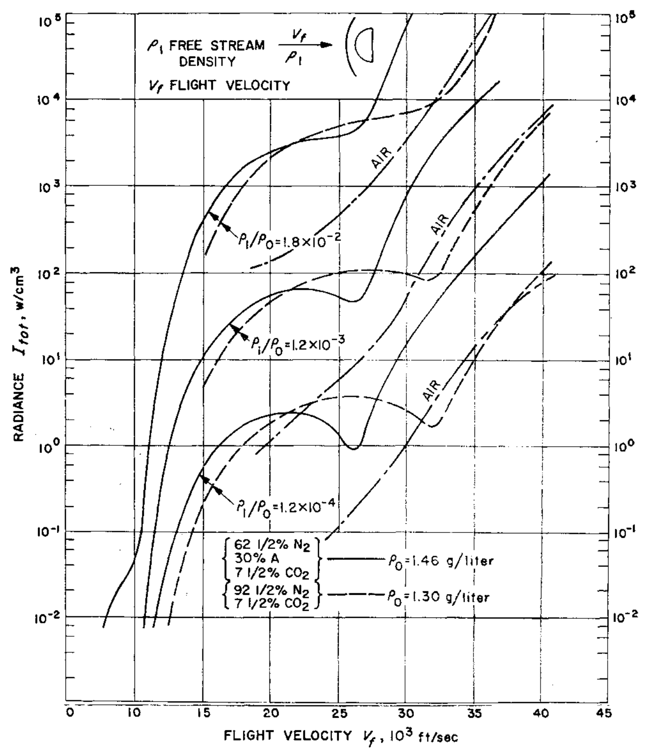

Fig. 2 Radiance of N_2, CO_2, and A mixtures.

celeration and, as can be seen from Fig. 2, the total radiance of the CO_2-N_2 mixture is fairly close to the values for air under the same freestream condition. The emissivity of argon which was insignificant at the lower flight velocities equals the radiance of the nitrogen that it replaces at 10,000°K and is radiating even more intensely at the higher speeds. However, the higher equilibrium temperature of the argon-bearing mixture dominates all other effects of chemical composition of the atmosphere, as indicated by the large increases in the total radiant intensity of the gas.

The effect of a varying CO_2 content (by volume) in the original two-component mixture is shown in Fig. 3, where the computed radiance is plotted for several flight velocities and compared to experimental points obtained in the Ames free-flight shock tunnel.[2] Changes in the total gas radiance due to a 30% argon addition replacing the same amount of nitrogen in the mixture are indicated by the arrows. At the lower velocities, the effect of argon addition is most noticeable in the 30% CO_2 mixture, which could generally move the maximum of the radiance to higher CO_2 percentages.

Since the effect of 30% argon addition is roughly the same for all gas mixtures considered, it must be concluded from these calculations that entry velocities to Mars should be kept below about 28,000 fps in order to avoid an order-of-magnitude increase in radiance, when large amounts of argon are presumed to be present. Below this velocity, the effect of argon addition is comparatively small, because of compensating effects.

2. Effect of Heat of Formation of CN

Within a flight velocity range from 15,000 to 30,000 fps, the radiance of mixtures of CO_2, N_2, and A is dominated by the energy emitted by the CN radical. The number of these particles is somewhat conjectural because of the uncertainty in the determination of its energy of dissociation. Values listed in different sources vary from 7.50 to 8.48 ev.[7,8] In addition, the distribution of radiated energy between the main spectral regions, the red and the violet band systems, also is not definitely established. Whereas older sources[4] used equal electronic oscillator strength for both, Fairbairn's data[9] show f-numbers for the violet band system to be five times larger than for the red. Experimentally, f-numbers can be derived from measurement of radiative decay times[10] or spectral intensity observations in shock tubes containing a CO_2-N_2 mixture.[9] In the latter case, the amount of energy is proportional to both number density and oscillator strength; therefore, keeping $(N_i f)$ constant, f-numbers vary inversely with the assumed number density of the radiating molecules.

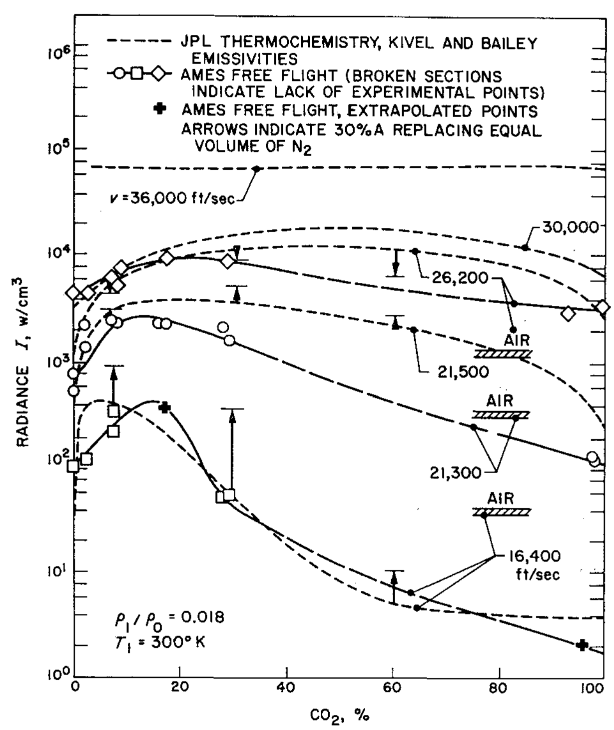

Fig. 3 Effect of argon addition on radiance of CO_2-N_2 mixtures.

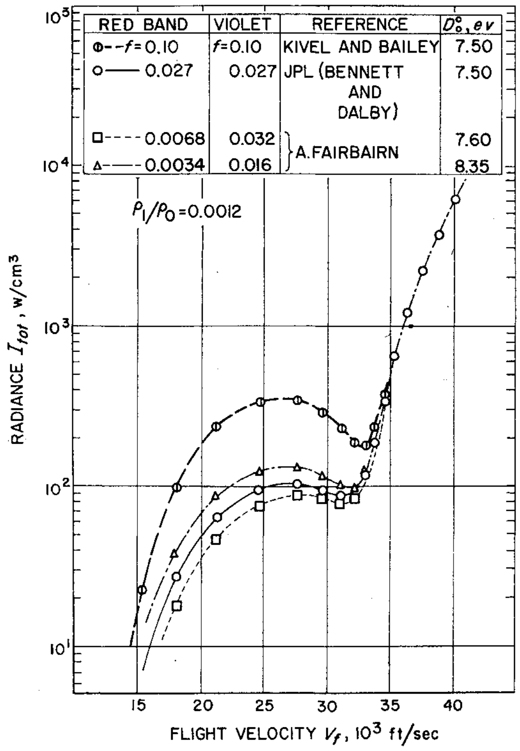

Fig. 4 Effect of CN-f-number variation on radiance of $7\frac{1}{2}\%$ CO_2-$92\frac{1}{2}\%$ N_2 mixture.

A comparison of the effect of changes in CN concentration and f-number values from different sources is shown in Fig. 4, where total radiant intensity of a $7\frac{1}{2}$ CO_2-$92\frac{1}{2}$ N_2 mixture at 0.0012 standard freestream density is shown as function of flight velocity, as obtained from the Jet Propulsion Laboratory thermochemistry normal shock computer program. It appears that all of the recent data on oscillator strength would result in shocked gas radiation within a $\pm 25\%$ tolerance band below approximately 30,000 fps. Above 32,000 fps, the contribution of the CN radical to the total emitted energy drops quickly because of the dissociation of CN and the appearance of electron acceleration and capture by atoms and ions as the principal radiation process.

3. Concluding Remarks

The choice of parameters used to describe the gas radiance from a shock layer (e.g., Figs. 2 and 3) is of importance to the proper application of these data to space-vehicle design.

Specifically, application of radiation data, such as the ballistic range data of Ref. 2, to Martian atmospheric entry must be done on the basis of matching two state conditions in the shock layer for a given initial gas mixture. Noting that, for hypersonic flow, the stagnation pressure is approximately twice the dynamic pressure, it is immediately apparent that freestream density is the controlling parameter for matching stagnation conditions. Variations of freestream temperature will have little effect, since the associated energy is small compared to the kinetic energy of the entry vehicle. In contrast, matching of freestream pressure, when the Martian ambient temperature can be less than half that in a ballistic range, will result in underestimating the radiation heat transfer by 150% in some cases.

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Heat Transfer on Power Law Bodies

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Nomenclature

δ = boundary-layer thickness
τ = shear stress
r = radius
P = pressure
ρ = density
u = velocity parallel to surface
μ = viscosity
X = coordinate distance measured along body
y = coordinate distance measured normal to body
M = Mach number
γ = ratio of specific heats
F = a constant
n = exponent of power body
l = Prandtl mixing length
D = e^{-FK}
a = $KU_\infty (\tau_\omega/\rho_\omega)^{1/2}$
α = $(2A^2 - B)/(B^2 + 4A^2)^{1/2}$
β = $B/(B^2 + 4A^2)^{1/2}$
A = $[(\gamma - 1)/2]M_\infty/(\tau_\omega/\tau_\infty)$
B = $(\{1 + [(\gamma - 1)/2]M_\infty^2\}/(\tau_\omega/\tau_\infty)) - 1$
K = l/y

Subscripts

ω = wall
∞ = edge of boundary layer or freestream
A = axially symmetric
$2D$ = two-dimensional

Introduction

THE development of an expression for the ratio of two-dimensional to axially symmetric shear stress in laminar flow for bodies of the form $r = cx^n$ was shown in Ref. 1; the result was

$$(\tau_A/\tau_{2D}) = (1 + 2n)^{1/2}$$

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Fully developed turbulent flow is considered in the current note; an expression similar to that for laminar flow is derived:

$$(\tau_A/\tau_{2D}) = (1 + n)^{1/5}$$

Discussion

The von Kármán momentum equation

$$\frac{\partial}{\partial X}\int_0^\delta r\rho u\,(u_\infty - u)dy = -\frac{\partial P}{\partial X}\int_0^\delta r dy - r_\omega\tau_\omega \quad (1)$$

can be reduced to

$$\frac{\partial}{\partial X}\int_0^\delta \rho u(u_\infty - u)dy + \frac{1}{r_\omega}\frac{\partial_{r_\omega}}{\partial X}\int_0^\delta \rho u\,(u_\infty - u)dy = -\tau_\omega \quad (2)$$

by assuming that the boundary-layer thickness is thin compared to the body dimension ($\delta \ll r$) and that $\partial P/\partial X = 0$. Reference 2 shows that for a flat plate [where $(1/r_\omega)(\partial_{r_\omega}/\partial X) = 0$]

$$\frac{\rho_\omega u_\infty}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}a^2\frac{d}{dX}\times \left\{\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} \quad (3)$$

Equation (3) was integrated to

$$\frac{\rho_\omega u_\infty X}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}a^2\times \exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right] \quad (4)$$

The introduction of $1/X = (1/r\omega)(\partial r\omega/\partial X)$ (for cones) in Eq (2) produced a new term in (3):

$$\frac{\rho_\omega u_\infty}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}\times \left(\frac{d}{dX}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} + \frac{1}{X}\left\{a^2\exp\left[\frac{\alpha}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}\right) \quad (5)$$

Equation (5) was integrated to

$$\frac{1}{2}\frac{\rho_\omega u_\infty X}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}a^2\times \exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right] \quad (6)$$

The current effort is an extension to Ref. 2 for more general body shapes described by the equation $r = cx^n$; $(1/r_\omega)\cdot(\partial_{r_\omega}/\partial X) = n/X$ can be substituted into Eq. (2), thereby changing Eq. (5) to

$$\frac{\rho_\omega u_\infty}{\mu_\omega} = \frac{D}{K^3(1 + B - A^2)^{1/2}}\times \left(\frac{d}{dX}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\} + \frac{n}{X}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}\right) \quad (7)$$

This is a differential equation of the form

$$\Delta' + (n/X)\Delta = \Lambda \quad (8)$$

where

$$\Delta' = \frac{d}{dX}\left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}$$

$$\Delta = \left\{a^2\exp\left[\frac{a}{A}\,(\sin^{-1}\alpha + \sin^{-1}\beta)\right]\right\}$$

$$\Lambda = \frac{\rho_\omega u_\infty}{\mu_\omega}\frac{K^3(1 + B - A^2)^{1/2}}{D}$$